4/26/18 "Too many of us are not living our dreams because we are living our fears."-Les Brown

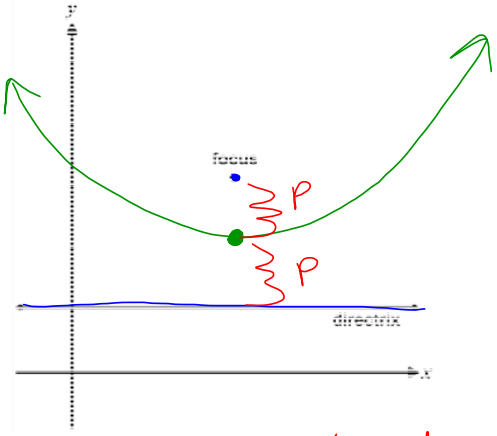
HW: "2017 A2 CC1 More Parabolas" HW Section
Test 1 on Wednesday 5/2

AIM: More Parabolas

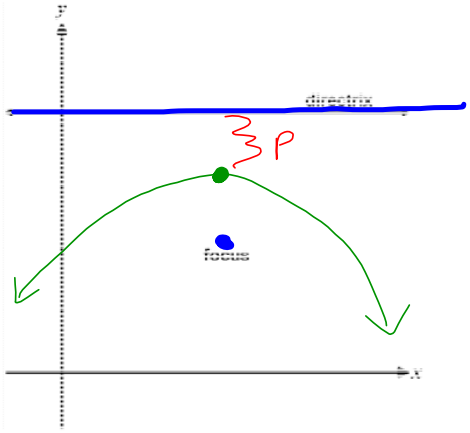
Warm Up:

***Exercise* #1:** Given the two scenarios below showing a focus and directrix, sketch the general shape of the parabola that would result. Place a point at the turning point of the parabola.

(a)



(b)



p is distance between focus and vertex

(c) If each of these parabolas had an equation written in its vertex form, i.e. $y = a(x-h)^2 + k$, would the leading coefficient, a, be positive or negative. State an answer for each and explain.

a) a would be positive b/c (focus above directrix)

b) a would be negative b/c (focus is below directrix)

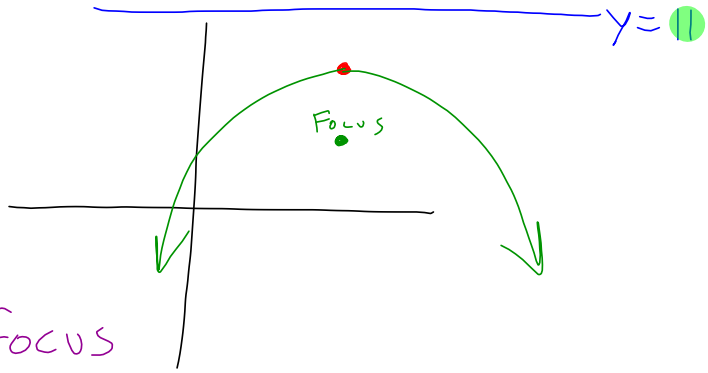
FOCAL LENGTH FORM OF A PARABOLA

If (h,k) are the coordinates of the vertex of a parabola and p is the distance from the vertex to the focus (known as the **focal length**) then the equation of the parabola is:

$\frac{(x-h)^2}{4p}+k = y = (\pm)\frac{1}{4p}(x-h)^2+k$ (where the sign depends on the orientation of the parabola $\cup$ or $\cap$)

+ −

Exercise #2: A parabola has a focus at $(5,3)$ and a directrix whose equation is $y=11$. Sketch a picture of the parabola and determine its equation using the form above.



Take y-values 3 and 11

$\frac{3+11}{2}=\frac{14}{2}=7$ Vertex $(5,7)$

To find p
Subtract y-values and then divide by 2

$\frac{11-3}{2}=\frac{8}{2}=4$

focus is below so the sign will be (−)

vertex: $(5,7)$ $p=4$

$$y=-\frac{1}{4(4)}(x-5)^2+7$$

$$y=-\frac{1}{16}(x-5)^2+7$$

$$y = \pm \frac{1}{4p}(x-h)^2 + k$$

Exercise #3: If a parabola has a directrix with equation $y = 18$ and a focus at $(4, 8)$ then the leading coefficient of the parabola would be which of the following?

(1) $-\frac{1}{40}$

(2) $-\frac{1}{20}$ Focus is below

(3) $\frac{1}{40}$

(4) $\frac{1}{20}$

$-\frac{1}{4(5)} = \frac{-1}{20}$

$y = 18$

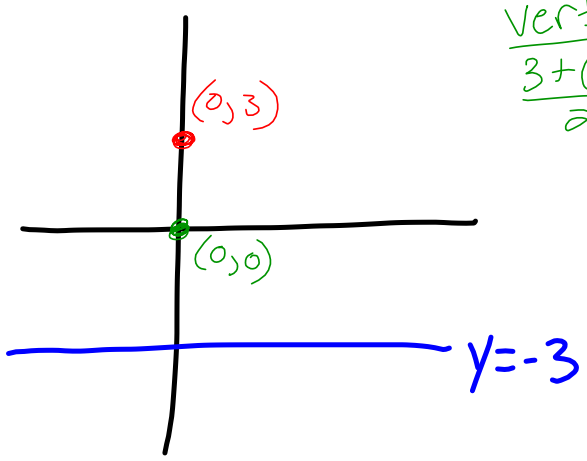
$(4, 8)$

$\frac{18-8}{2} = 5 = p$

Exercise #4: A parabola has a focus at $(0, 3)$ and a directrix whose equation is $y = -3$.

(a) Sketch a graph of the parabola on the axes below, determine the focal length, f, and the equation of the parabola using the focal length formula.

(b) Use the distance formula to derive the equation of the parabola based on the locus definition.



Vertex:

$\frac{3+(-3)}{2} = 0$

$(0, 0)$

p-value

$\frac{3-(-3)}{2} = \frac{6}{2} = 3$

$p = 3$

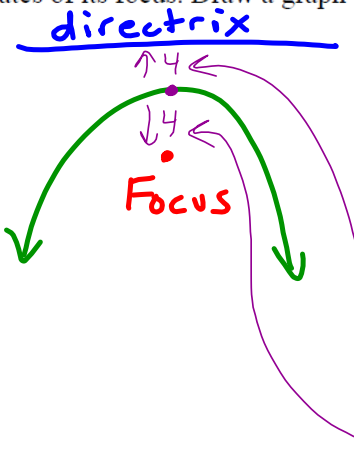
Focus is above:

$$y = +\frac{1}{4(3)}(x-0)^2 + 0$$

$$y = \frac{1}{12}x^2$$

$y = \pm \frac{1}{4p}(x-h)^2 + k$

***Exercise* #5:** A parabola has the equation $y = -\frac{1}{16}(x-2)^2 + 6$. Determine the equation of its directrix and the coordinates of its focus. Draw a graph to justify your answers.



$-\frac{1}{16} = \frac{1}{4p}$

$\frac{16}{4} = \frac{4p}{4}$

$4 = p$

directrix $y = 10$

4

vertex $= (2, 6)$

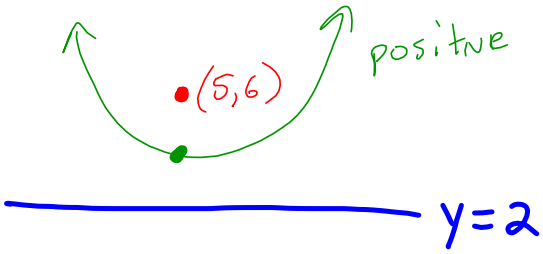
-4

Focus $(2, 2)$

HW Section:

1) $F=(5,6)$

$D=y=2$



Need:

Vertex:

Add y-values of focus and directrix and divide by 2 to get y-value of vertex. x-value is same as focus.

$\frac{2+6}{2}=4$ ← y-value

5 ← x-value

Vertex $=(5,4)$

P-value:

Subtract y-valves from focus and directrix, then divide by 2

$\frac{6-2}{2}=\boxed{2=p}$

$y=\pm\frac{1}{4p}(x-h)^2+k$

$\boxed{y=+\frac{1}{4(2)}(x-5)^2+4}$

Review:

4) $x^2 + 10x - 4y + 9 = 0$

$x^2 + 10x + 9 = 4y$

$x^2 + 10x + \boxed{25} + 9 - \boxed{25} = 4y$

$\frac{10}{2} = 5$

$5^2 = 25$

$\frac{1}{4}(x+5)^2 - 16 = \frac{1}{4}(4y)$

$\frac{1}{4}(x+5)^2 - 4 = y$

Vertex: $(-5, -4)$

$+1$

Focus: $(-5, -3)$

$\frac{1}{4} = \frac{1}{4p}$

$4 = 4p$

$1 = p$